I. INTRODUCTION

Focusing x-ray spectrographs of Johann, Johansson, and Cauchois type are successfully used in plasma diagnostics.\(^1\)\(^,\)\(^2\) The dispersive elements of these devices are usually quartz, mica, and other crystals, bent cylindrically, spherically, or toroidally. The reflecting crystal planes are parallel to the mechanical crystal surface in Johann geometry, but perpendicular to the surface in Cauchois geometry. Johann and Johansson devices use a reflection geometry appropriate for the softer x rays with photon energy \(E<10\text{ keV}\). These devices need to be in vacuum when \(E<5\text{ keV}\) to prevent the absorption of x rays by air. Cauchois devices use a transmission geometry. They are employed when \(E>10\text{ keV}\) and do not need vacuum. These differing requirements differentiate the design of transmission devices from the ones of reflection type.

This article describes a high-resolution x-ray spectrograph that combines reflection and transmission geometries. The two principal novelties of this device are the Cauchois–Johansson transmission geometry, described later in detail, and the dispersive element. This is a specially designed quartz crystal glued to a cylindrical substrate. The device’s vacuum envelope makes the device suitable even for the softest x rays, and allows a wide range of incident angles, 25–75 degrees. Spectra from the x-ray tube are presented, which provides high spectral resolution in transmission and reflection regimes and covers the wide energy range of 1.5–400 keV. The device is calibrated with an x-ray tube and is suggested to be used in high-temperature plasma diagnostics. © 2001 American Institute of Physics. [DOI: 10.1063/1.1324754]

II. CAUCHOIS–JOHANSSON GEOMETRY AND COMPARISON WITH OTHER TECHNIQUES

We briefly remind the reader here of the traditional Johann, Johansson, and Cauchois optical schemes. Then we describe the Cauchois–Johansson optical scheme, with an emphasis on what is new in comparison with existing instrumentation.

Figure 1(a) shows the optical scheme of the Johann device\(^3\) when measuring the x-ray spectrum emitted by a point source. For simplicity we consider the case of symmetric diffraction from the middle plane of the crystal, and we put the point x-ray source \(A\) just on the Rowland circle. X rays are reflected from the crystal \(K\) according to Bragg’s law \(2d \sin \theta = k \lambda\), where \(2d\) is twice the interplanar crystal distance, \(k\) is the order of reflection, and \(\lambda\) is the wavelength. The spectra are focused on a Rowland circle with radius \(R\).

In the Johann device the radius of the reflecting surfaces of crystal \(K\) is equal to \(2R\), and the radius of the mechanical crystal surface is \(2R\). If the source is just on the Rowland circle, any given wavelength is reflected by all the crystal surfaces. Even if the crystal is perfect and we only consider x rays reflected by the crystal’s central planes, the spectral line at the detector position is slightly defocused: \(B_1\) and \(B_2\) do not coincide. This kind of geometrical defocusing is caused by the influence of crystal edges, which lie slightly above the Rowland circle. From the geometry it follows, \(\arccos AB_2 = \arccos AB_1\), \(\arccos AB_3 = 2R \theta\), \(\theta = (\arccos AB_2 - \arccos CD)/2R\), and \(\arccos B_1 B_2 = \arccos CD\).

In the Johansson device\(^4\) [see Fig. 1(b)], the radius of the crystal reflecting planes is \(2R\), but the radius of the mechanical crystal surface is \(R\). Now the x rays are focused exactly on the detector position: \(B_1\) coincides with \(B_2\) as shown in Fig. 1(b). \(\arccos AB_1 = \arccos AB_2 = 2R \theta\) identically, and \(\arccos B_1 B_2 = 0\). However, in practice even the best flat crystals are not ideal, and even a single x-ray energy is reflected over a finite angle, the width of the so-called rocking curve. This width is the smallest for a perfect flat crystal. Unfortunately, bending the crystal can double or even quadruple the rocking width of the rocking curve.
metrical considerations

5 Most x rays coming from the extended source happen in Johann devices, but the defocusing from the crystal’s curvature and size is always less in the Johansson scheme than in the Johann scheme for identical crystal cuts and whether the crystal is bent cylindrically, spherically, or toroidally.

Figure 2(a) presents the optical scheme of the Cauchois device. Most x rays coming from the extended source \( A_1 A_2 \) go through the crystal directly, but some are reflected according to Bragg’s law. Spectra are focused on the Rowland circle with radius \( R \) when the radius of the mechanical crystal surface is equal to \( 2R \). The crystal reflecting planes are perpendicular to the mechanical crystal surface: the reflecting planes would cross at the same point on the Rowland circle if they were to be extended that far. Focusing of spectral lines in this scheme is not ideal because the crystal edges are not exactly in the right position (geometrical component) and because of diffraction divergence. In this sense the Cauchois optical scheme is similar to the Johann geometry, and we call it here the Cauchois–Johann scheme.

Figure 3(a) illustrates the geometrical defocusing of a spectral line in Cauchois–Johann devices. Point \( C \) is the center of the crystal, \( A \) is a point on the crystal, and \( CA \) is the crystal length reflecting a given wavelength \( \lambda \) at the Bragg angle \( \theta \). \( D'E' = DE \) is the geometrical defocusing of the line. The angle \( \gamma = \arccos \frac{AC}{2R} \) is a measure of the crystal size, while \( \arccos \frac{DE}{\alpha R} \) measures the line width. From geometrical considerations

\[
\begin{align*}
\alpha &= \gamma - \theta + \arcsin \left(\sqrt{5-4 \cos \gamma \sin (\theta - \beta)}\right), \\
\sin \beta &= \sin \gamma / \sqrt{5-4 \cos \gamma}
\end{align*}
\]

and for small \( \gamma \) we obtain

\[
\begin{align*}
\Delta \theta_{\text{geom}} &= \frac{1}{2} (\arcsin(5-4 \cos \gamma)^{1/2} \sin (\theta - \beta) + \gamma - \theta) \\
&= (\arcsin(5-4 \cos \gamma)^{1/2} \tan (\theta - \gamma) + \gamma - \theta) \\
&= (\gamma^{1/2} \tan (\theta - \gamma) + \gamma - \theta).
\end{align*}
\]

In analogy with this we estimate the geometrical defocusing for a reflection Johann device:

\[
\begin{align*}
\Delta \theta_{\text{geom}} &= \frac{1}{2} (\arcsin(5-4 \cos \gamma)^{1/2} \cos (\theta - \beta) + \theta - \gamma - \pi/2) \\
&= (\arcsin(5-4 \cos \gamma)^{1/2} \cot (\theta - \gamma) - \gamma - \theta) \\
&= -(\gamma^{1/2} \cot (\theta - \gamma) - \gamma - \theta).
\end{align*}
\]

However, except for geometrical and diffraction defocusing, the device with the bent crystal has the defocusing, caused by change of \( d \) after bending. This effect is shown on Fig. 3(b) for a transmission-type crystal. We call \( h \) the thickness of the crystal, \( d, (d+\Delta d/2), \) and \( (d-\Delta d/2) \) the intermediate crystal distance in the middle, outer and inner cross sections respectively, \( \Delta d = \alpha h, \) and \( R_c \) the radius of the crystal. From geometry it follows

\[
\Delta d = dh/R_c.
\]

Derivating Bragg equation \( 2d \sin \theta = \lambda \) under the condition \( \partial \lambda = 0 \) one can obtain

\[
\Delta d = (\Delta d/d) \tan \theta.
\]
From (6) and (7) we obtain

$$\Delta \theta_d = (h/R_{cr}) \tan \theta \approx (h/R_{cr}) \theta. \tag{8}$$

Let us calculate the line width $DE$ for $R = 250$ mm and $	heta = 30^\circ$ for a Cauchois–Johann device. The typical length of the crystal is 50–70 mm and in the case of extended source the entire crystal surface reflects the line. Therefore the crystals subtends an angle $\gamma = 5^\circ$. Using (1) we obtain the geometrical defocusing angle $\alpha = 0.004$ rad, so that the line on the detector is $DE = \alpha R \approx 1$ mm wide. This geometrical widening of the line can also reflect an extended source, a broad line, or a point source that moves during exposure. The corresponding geometrical component of resolution $(\Delta \lambda / \lambda)_{geom} = \gamma/2 \approx 4 \times 10^{-3}$; it does not depend on $\theta$. The resolution component because of change in $d$ is $(\Delta \lambda / \lambda)_d = \Delta \theta_d \cot \theta = h/R_{cr} \approx 0.7 \times 10^{-3}$, which also does not depend on $\theta$.

Let us compare the geometrical resolution with an estimated diffraction component. In the best case $\Delta \theta_{diff} \approx 50$ arcsec for a bent crystal of 250 mm radius. Then $(\Delta \lambda / \lambda)_{diff} = (\cot \theta) \Delta \theta_{diff} = 0.4 \times 10^{-3}$, less than the geometrical component. It is a nontrivial problem to decrease $\Delta \theta_{diff}$, because this strongly depends on the mechanical distortion of the bent crystal.

We propose the optical scheme in Fig. 2(b) to decrease the geometrical defocusing of lines in transmission geometry. In Fig. 2(b) the mechanical radius of the crystal surface is now equal to $R$, as in the Johansson scheme. Geometrical continuations of the reflecting planes cross in the same point on the Rowland circle, just as in the Cauchois–Johann scheme. Taking into account these points we call this scheme the Cauchois–Johansson one. In analogy with the Johann and Johansson schemes, the spectral resolution of a Cauchois–Johansson device should be much better than that of a Cauchois–Johann device. It is clearly seen from Fig. 2(b) that $AB = 2R \theta$ identically for any arbitrary angle and for all the points along the crystal length. Therefore, the line width does not depend on the source size, and the position of the line does not depend on the position of the source. Geometrical defocusing is zero for the middle plane of the crystal. As always, this statement applies only to an ideal crystal. Resolution depends on the diffraction defocusing of the line and defocusing because of change in $d$. The art is now to manufacture a crystal that is as close to ideal as humanly possible.

To realize the Cauchois–Johansson scheme we designed and manufactured a quartz crystal 70 mm in length and 10 mm in width, and glued this crystal to a cylindrical substrate with a radius of 250 mm. The crystal can be applied in two ways [see Fig. 4(a)]: with an intermediate distance for symmetric transmission scheme, $d = 1.8$ A (0001 cut), and with $d = 2.457$ A (11–20 cut). An intermediate distance for a sym-
metrical reflection scheme is $d = 4.255 \text{ Å (10-10 cut)}$ for both cases. This crystal has the following advantages:

(i) It works in transmission Cauchois–Johanna geometry, shown in Fig. 2(b), with the main cut 0001 or 11(−2)0, and also with a series of asymmetric cuts, given in Figs. 7–10.

(ii) It works in reflection in the Johansson geometry, shown in Fig. 2(b), with the main cut 10-10 and the series of asymmetric cuts given in Fig. 6.

(iii) When the crystal is manufactured with high accuracy, the position of the line does not depend on the position of the source and the width of the line does not depend on the source size for both transmission and reflection schemes.

Figure 4(b) shows the optical scheme with combined crystal. A is an x-ray source, B is the focused line for Johansson geometry, and $A_1A_2$ is an x-ray source for the Cauchois–Johann geometry.

Although the manufacturing process of the required crystal is technologically more challenging than that for the simpler Cauchois–Johann crystal, we should mention that similar combined dispersive elements can be done only from quartz, which has high elasticity, mechanical strength, and abundance of cuts with high reflection coefficients.

III. ENERGY RANGE OF THE DEVICE

We analyzed the energy range of the device with the combined Cauchois–Johann crystal in reflection Johann scheme, where the symmetric cut of our crystal is 10(−1)0. Figure 5(a) illustrates the symmetric and asymmetric cuts in the reflection scheme. The angle $\alpha$ is the angle between the given cut and tangent to Rowland circle in point O. For symmetric cuts $\alpha = 0^\circ$. Let us call the angle $\varphi$ the incident angle with the crystal surface and $\phi$ the reflected angle with the crystal surface. Then $\varphi = \theta \pm \delta$, $\phi = \theta \mp \alpha$.

Figure 6 shows the corresponding plots of energy versus Bragg angle for our crystal, working in Johansson geometry. Energy range of the device in Johansson geometry is 1.5–13.2 keV for angle range 25°–75° and for the series of cuts 10(−1)1, 20(−2)1, 30(−3)1, 40(−4)1, 50(−5)1, and 50(−5)2.

Figure 5(b) shows the zero orders for reflection in the transmission scheme for the Cauchois–Johann device with a crystal with asymmetric cuts. The zero order is the detector coordinate, corresponding to $\lambda = 0$. We mean that the zero order for the symmetric cut is point A. Each asymmetric cut has its mirror twin due to the inherent symmetry of the quartz crystal. Therefore, left and right zero orders exist for each cut and we show here right and left orders of reflection. The given line can be registered in both orders, if the corresponding incident and reflected angles are accommodated by the device’s vacuum envelope and other experimental conditions.

Figures 7 and 8 show the plots of energy versus the Bragg angle for right and left orders for 0001 cut and corresponding asymmetric cuts in transmission geometry. Figures 9 and 10 give the same plots for 11(−2)0 cut. From those figures the energy range of the device in transmission geometry for 25°–75° angle range is 1.7–400 keV. However, the

![FIG. 9. Energy versus Bragg angle for right orders of 11(−2)0 cut and very near asymmetric cuts in transmission geometry. Angle range 25–75 degrees.](image)

![FIG. 10. Energy versus Bragg angle for left orders of 11(−2)0 cut and very near asymmetric cuts in transmission geometry. Angle range 25–75 degrees.](image)

![FIG. 11. (a) Photo of device and (b) cylindrical combined crystal and spherical Johansson crystals on optical contact.](image)

![FIG. 12. (a) Cr Ka lines, taken in second order of reflection from 10(−1)0 cut. (b) Mo Ka lines, taken in the transmission regime in second order of reflection from 10(−1)0 cut and in first order from 13(−4)0 cut. Cu Ka lines taken in transmission regime in first order of reflection from 14(−5)0 cut.](image)
x-ray energy on the low side is limited by absorption inside the crystal. At energies greater than 400 keV spectral resolution is unsatisfied. Therefore, for this device with 0.35 mm thickness crystal, we state a 10–400 keV energy range for the transmission geometry. The total energy range of the device, equipped with our combined crystal, is 1.5–400 keV.

The detailed analysis of the device, given in this section, is important for interpretation of complicated spectra for the choice of the cut with optimal combination of resolution and light power to register lines with the given wavelength and brightness. A similar device is suited for absolutely calibrated measurements of Bremsstrahlung,8 but in this case it is necessary to use cuts without overlapping of the reflection orders. If the device is calibrated in wavelengths, the heavy ion’s velocity can be determined from the Doppler shift.

IV. CALIBRATION OF DEVICE

We used a standard x-ray tube with current I=40 μA and voltage U=30 kV to verify the operation of the combined Cauchois–Johansson crystal with 11(−2)0, d = 2.457Å symmetric transmission cut and 10(−1)0, d = 4.255Å symmetric reflection cut. Figure 12(a) shows the CrKα spectrum, reflected from the 10(−1)0 cut in the reflection geometry of Fig. 1(b). Lines are visible at 5.4 keV, separated by 0.09 keV, superposed on the bremsstrahlung continuum. Figure 12(b) shows a MoKα spectrum at 17.4 keV, separated by 0.105 keV, taken in the transmission geometry of Fig. 2(b). Those spectra are obtained with 13(−4)0 cut and second order of 10(−1)0 cut. This much harder spectrum is also obtained with good precision. The two K-lines are clearly seen at the expected energies of 17.37 and 17.48 keV. We also observed fluorescent CuKα lines, 8.047 keV and 8.027 keV, emitted by the border of the x-ray tube and reflected from 14(−5)0 cut. These Cu lines are well resolved, inspite of the fact that the source size is about 7 mm. These experimental results demonstrate that one single crystal can work in transmission and reflection regimes, providing high resolution in both the schemes.

V. CONCLUSIONS

(1) We propose a Cauchois–Johansson optical scheme, which provides higher spectral resolution in the transmission regime than that of the Cauchois-Johann-type scheme.

(2) We have manufactured an x-ray spectrograph with especially designed cylindrical quartz crystal. The crystal works in transmission Cauchois–Johansson geometry with d = 1.8Å, cut 0001, or d = 2.457Å, cut 11(−2)0. The same crystal works in reflection Johann geometry with d = 4.25Å, 10(−1)0.

(3) The energy range of the device was analyzed with accounting of symmetric and asymmetric cuts. The 1.5–400 keV energy range is achieved, which is wider than that of existing prototypes.

(4) Calibration, made with an x-ray tube, showed that one crystal can work in transmission and reflection regimes, providing high spectral resolution in both the schemes. The device is suggested to be used in high-temperature plasma diagnostics.


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