Mass loss of exploding foils

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A thin asymmetrically driven exploding foil loses mass when the acceleration of the bulk of the foils exceeds the acceleration of the foil's back. A simplified but exactly solvable model clarifies the mass erosion process.

Opening a current-carrying circuit by quickly separating metallic conductors works well in low-power applications. In high-power circuits the magnetic pressure may be sufficient to move thin conductors during the current pulse, and in this case a moving foil could be used to break a circuit.

Experimental evaluation[1,2] of a moving foil as a circuit breaker was moderately successful when the circuit has a low inductance, interrupting about half the input current. The low efficiency was not unexpected, however, it was surprising that the interruption occurred earlier than calculated from Newton's law using the magnetic pressure and the foil mass per unit area. An earlier paper[3] identified and evaluated one mechanism for early breaking of the circuit, viz., mass loss from the bulk of the foil due to differential acceleration.

Typically the resistance in a metallic foil increases with temperature, and the current density decreases where the foil becomes hot. In the situation suggested by Fig. 1 the back of the foil, at $x=0$, is toward the side of the input current; the foil front, at $x=-d$, connects to an output circuit. Then the magnetic force density in the foil can become largest somewhere inside rather than on the back. In the absence of material strength the foil rips apart. The back of the foil stays behind, while the front part splits off, accelerating faster than if the foil were intact.

Assuming constant density ρ and ignoring material strength the foil splits at a point x_s where the local acceleration becomes less than the acceleration of the foil in front of this point. In the front part of the foil the local acceleration increases from $x=-d$ going into the foil, and the foil moves as a solid. Its acceleration is the difference in magnetic pressure $\Delta(\mu_0 H^2/2)$ between the foil edge at $x=-d$ and x_s, divided by the mass per unit area $\int_{-d}^{x_s} dx\,\rho$ of this part of the foil. The local acceleration is given by the gradient in the magnetic pressure, $-\nabla(\mu_0 H^2/2)$, divided by the local mass density ρ. Thus, the foil splits when

$$-\frac{1}{\rho}\frac{\partial(\mu_0 H^2/2)}{\partial x} \leqslant \frac{[\mu_0 H^2(-d)-\mu_0 H^2(x_s)]/2}{\int_{-d}^{x_s} dx\,\rho}. \tag{1}$$

This equation determines the splitting position x_s; mass loss starts when x_s reaches the foil's back, i.e., when $x_s=0$.

An extreme example is a foil that suddenly changes from a conductor into an insulator, e.g., when the magnetic field exceeds a certain value. The rear of the foil starts insulating first. No current flows in the insulating part of the foil, the force density vanishes, and the insulating material coasts along. In contrast, the front part of our idealized foil still accelerates, and it will put away from the insulating fraction.

It should be emphasized that the behavior of thin foils in such situations is influenced by many other effects (e.g., kinetic pressure from hot plasma, nonuniform electrical breakdown, etc.) that may be more important than the mass erosion phenomenon discussed. Nevertheless, mass erosion is certainly a contributor to earlier opening of the foil opening switch.

To gain analytical insight into the splitting process, this communication uses the magnetic fields from a special solution to the magnetic diffusion equations. The relevant component of the magnetic field H (or the current density j) penetrates into our one-dimensional foil according to

$$\frac{\partial E}{\partial x}=\mu_0\frac{\partial H}{\partial t}, \tag{2a}$$

$$\frac{\partial H}{\partial x}=j, \tag{2b}$$

where E is the relevant component of the electric field. The current heats the foil; without thermal diffusion the energy per unit mass Q increases according to

$$\frac{\partial Q}{\partial t}=jE. \tag{2c}$$

The current density is given by the electric field and the resistivity η, but as the material heats up the resistivity increases, and

$$E=\eta(Q)\,j. \tag{2d}$$

Often the resistivity is proportional to the heat content Q; starting at the melting point where $Q=Q_m$, the resistivity

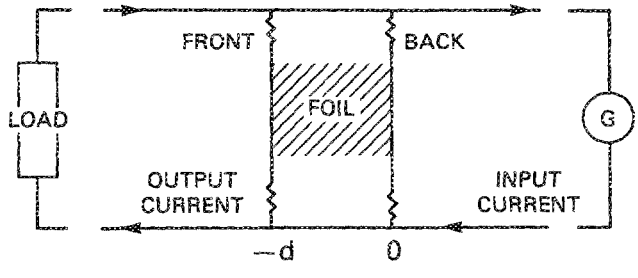

FIG. 1. Foil geometry. The input current comes from the left at the foil's backside at $x=0$, and the output current is on the right at the foil's front side at $x=-d$.

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increases more rapidly. At the boiling point and beyond the resistivity is difficult to characterize.

As a function of normalized heat content $q = Q/Q_m$ the resistivity is

$$\eta(q) = \eta_0 f(q), \tag{3}$$

where the function $f(q)$ depends on the material. The simplest model[4] is a linear resistance increase, $f = 1 + \beta q$, with β the resistivity increase until melting. For room-temperature aluminum $\beta \sim 3.8$, and for copper $\beta \sim 5.5$. Beyond the melting point the resistance may still increase linearly, but with a larger value for the slope β_m, e.g., $\beta_m \sim 9$ for aluminum.

The next step is to define the boundary conditions on the magnetic field. On the backside the magnetic field is given by the input current $I(t)$ per unit length l, $H(t) = I(t)/l$. The magnetic field at the foil front and the output current are similarly related.

When the input and output currents are given, or when these are determined by an electrical circuit,[3] the partial differential equations (2) must be solved numerically. The alternative taken is to consider a case for which the partial differential equations simplify to ordinary differential equations. One choice is to use self-similar variables, but the appropriate choice is a magnetic wave with constant speed driven into the foil by a specific boundary condition. The sole argument in all field variables is now $x + vt$. These special solutions are well known[5]: the wave solution was used recently for a melting problem.[6]

The progressive wave solution works nicely for an infinite half-space, where a suitable input current specifies the boundary condition for the magnetic field at the back edge. In this case the other boundary condition is that all fields vanish for $x \to -\infty$. Then Eq. (2a) gives $E = \mu_0 v H$, and Eq. (2c) combined with Eq. (2b) integrates to $Q = \mu_0 H^2/2$.

The heat equation (2c) becomes

$$\partial q/\partial \zeta = q/f(q), \tag{4a}$$

where $\zeta = (x + vt)/\delta$, and the skin depth $\delta = \eta_0/2\mu_0 v$. Integrating gives

$$\zeta = \int_1^q \frac{f(x)dx}{x}. \tag{4b}$$

Here the value $\zeta = 0$ is chosen to be at the melting point, i.e., at $q = 1$.

Obviously, the shape of the magnetic wave depends on the resistivity (3). Figure 2 shows the heat content q, the (normalized) magnetic field $h(\zeta) \sim q^{1/2}$, and the normalized magnetic pressure $-dq/d\zeta$ for the progressive wave in aluminum, using $f = 1 + \beta q$ with $\beta = 3.8$ below melting at $q = 1$, and a break in slope to $\beta_m = 9$ for $q \geqslant 1$. Where the foil is still cold, $q \ll 1$, the resistivity is constant and as a consequence the diffusion is exponential. For $q \sim 1$ and larger the heat is linear in ζ, whence the magnetic field follows $\zeta^{1/2}$. The wave travels from right to left into the foil; the cross-hatched block suggests the wave's location in the foil at a particular time (before melting).

The magnetic field at the foil back $H_0(t)$ is given by the intersection of the wave with the foil boundary at $x = 0$ (the * in Fig. 2). This in turn defines how the input current

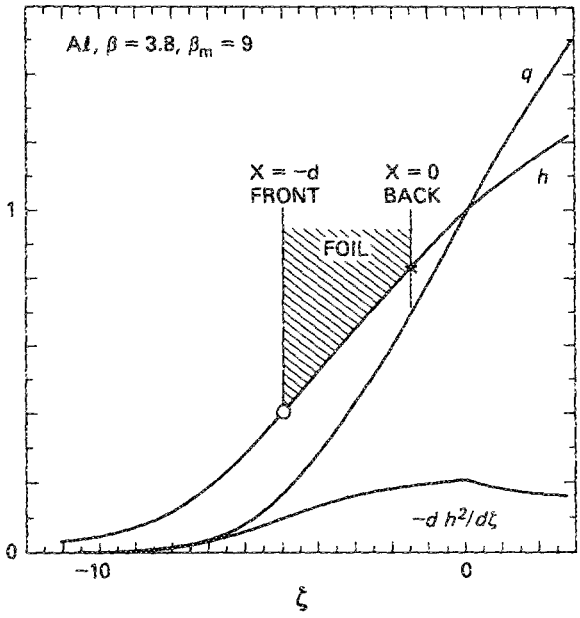

FIG. 2. The magnetic diffusion wave for an infinite slab of aluminum at room temperature or for a finite foil with judiciously chosen output current. Shown is the normalized heat content $q(\zeta)$, the normalized magnetic field $h(\zeta)$; the gradient of the magnetic pressure or the magnetic force density $-dh^2/d\zeta$ shows the effects of the more rapidly increasing resistivity at melting.

$H_0(t)l$ should behave in time to be appropriate for the progressive wave. Likewise, the output current at the front of the foil must be specified in accordance with the progressive wave (the O in Fig. 2). This way the magnetic diffusion wave never notices that the foil has a finite thickness. It may be impossible to get the correct currents in practice, but this detail is irrelevant for our discussion.

The change in time of the heat content $q_0(\tau)$ at the foil back, where $x = 0$ and $\zeta = vt/\delta = \tau$, follows from inversion of

$$\tau = \int_1^{q_0(\tau)} \frac{f(q)dq}{q}. \tag{5}$$

The heat at the back also determines the current per unit length, $I/l = (2qQ_m/\mu_0)^{1/2}$, or $I \propto q^{1/2}$. The penetration velocity v is determined by a characteristic risetime t_0 at the appropriate point in the input current pulse, viz., $v = (\eta_0/2t_0\mu_0)^{1/2}$.

Using the progressive wave solution the foil splitting becomes particularly easy because the heat content Q is equal to the magnetic pressure $\mu_0 H^2/2$. According to Eq. (1) the foils splits when $x_s = 0$, or in normalized form when

$$\frac{dq}{d\zeta}(\tau) = \frac{q(\tau) - q(\tau - d/\delta)}{d/\delta}; \tag{6a}$$

in terms of the resistivity function $f(q)$ this becomes

$$f[q(\tau)] = \frac{d/\delta}{1 - q(\tau - d/\delta)/q(\tau)}, \tag{6b}$$

after inserting Eqs. (4).

Figure 3 shows the function $f(q)$ for room-temperature aluminum; the straight lines meet at the melting point at $q = 1$. The curved lines are the right side of Eq. (6b) for four values of the foil thickness d normalized to skin depth δ, viz.,

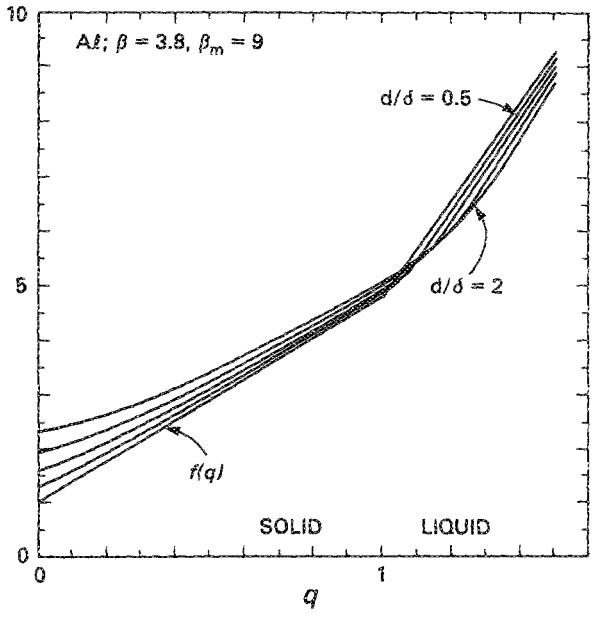

FIG. 3. The resistivity $\eta(q)$ for aluminum, and the right-hand side of Eq. (6b) for different foil thicknesses $d/\delta = 0.5$, 1.0, 1.5, and 2.0. The intersection of the curves defines where the foil splits.

$d/\delta = 0.5$, 1.0, 1.5, and 2.0. Splitting occurs where the curves cross. For all foil thicknesses the splitting starts increasingly beyond the melting point as the foil becomes thicker. Nevertheless, the melting point is a good approximation for all foils in this case, undoubtedly because the resistance increases so suddenly.

Equation (6) is hard to solve analytically because it involves the function $q(\zeta)$ at the two foil edges, i.e., at two different positions. However, when the foil is thin a Taylor expansion of $q(\tau - d/\delta)$ to second order in $d/\delta \ll 1$ around τ gives for Eq. (6)

$$\frac{f}{q} \lesssim \frac{df}{dq}. \tag{7}$$

This relation shows that thin foils will not split provided the resistivity increases linearly with heat content. Splitting will occur after a break in the resistivity at melting, $q = 1$, when the resistivity beyond melting increases linearly with a coefficient β_m larger than $\beta + 1$. This is certainly the case for aluminum, where $\beta + 1 \sim 4.8 < \beta_m \sim 9$.

Expanding to third order adds a correction term to Eq. (7), viz.,

$$\frac{f}{q} \lesssim \frac{df}{dq} + \frac{1}{3q}\frac{d}{\delta}\left[1 - \frac{3q}{f}\frac{df}{dq}\left(1 - \frac{q}{f}\frac{df}{dq}\right) - \frac{q^2}{f}\frac{d^2f}{dq^2}\right]. \tag{8}$$

At splitting, $1 - (f/q)df/dq \sim 0$ and the bracketed part of the correction term vanishes. When there is a sudden increase in the resistivity at melting, d^2f/dq^2 is large, and the remainder of the correction term in the $[\cdots]$ is negative; the thin foil is guaranteed to split.

Once the foil has begun to split, the mass on the left of the splitting point x_s disappears from the foil's backside, i.e., the mass loss is ρx_s, and the mass loss rate is $\rho\, \partial x_s/\partial t$. The splitting point x_s moves into the foil as $\partial x_s/\partial t$, which is $(\partial x_s/\partial q) \times (dq/d\tau)(\tau/t)$. Assuming that $\partial x_s/\partial q$ remains approximately constant, e.g., equal to the value where the foil starts to split $\partial x_s/\partial q \sim (\partial x_s/\partial q)(x_s = 0)$, the mass loss ρx_s follows by integration over time. Clearly the mass loss is proportional to the heat content Q, and equivalently to the magnetic pressure $\mu_0 H^2/2$. This was already found in previous numerical studies.[3]

The progressive magnetic wave illustrates that the foil splitting process is connected to the change of resistivity $\eta = \eta_0 f(q)$ with heat content q. Splitting tends to occur where the resistivity increases rapidly with heat content, as happens at melting. The mass loss is proportional to the magnetic pressure $\mu_0 H^2/2$. Qualitatively these conclusions should remain valid for more realistic situations; magnetic diffusion is often[7] well approximated by a progressive wave profile, even though the current only approximates the correct current rise [typically $\sin t/\tau$ vs $(t/\tau)^{1/2}$].

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